PROBLEM SET 2

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As before, submit the solution to Daniel via email by March 04, 2024.

Exercise 2.2. *Show that substitutions* $\Gamma \vdash \gamma : \Gamma.A$ *satisfying* $\mathbf{p} \circ \gamma = \mathbf{id}$ *are in bijection with terms* $\Gamma \vdash a : A$.

Exercise 2.4. *Given* $\Delta \vdash \gamma : \Gamma$ *and* $\Gamma \vdash A\ \mathsf{type}$*, construct a substitution that we will name* $\gamma.A$*, satisfying* $\Delta.A[\gamma] \vdash \gamma.A : \Gamma.A$.

Exercise 2.5. *Suppose that* $\Gamma \vdash A\ \mathsf{type}$ *and* $\vdash \Delta\ \mathsf{cx}$*. Show that substitutions* $\Delta \vdash \gamma : \Gamma.A$ *are in bijection with pairs of a substitution* $\Delta \vdash \gamma_0 : \Gamma$ *and a term* $\Delta \vdash a : A[\gamma_0]$.

The following is a bonus exercise; we encourage you to attempt it, but it is not mandatory to complete. For those categorically inclined, we encourage you to see if you can relate this property to a more familiar categorical statement.

Exercise 2.3. *Show the following equation* $(\gamma.a) \circ \delta = (\gamma \circ \delta).a[\delta]$.

Date: February 26, 2024.